

DOCUMENT RESUME

ED 469 079

SE 066 824

AUTHOR Rochowicz, John A., Jr.
TITLE Calculating the Variables of Finance on a Spreadsheet.
PUB DATE 2002-00-00
NOTE 21p.
PUB TYPE Reports - Descriptive (141)
EDRS PRICE EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS Algebra; Computer Uses in Education; Curriculum Development;
*Educational Technology; Equations (Mathematics); Estimation
(Mathematics); Higher Education; Instructional Design;
*Mathematical Applications; Mathematics Instruction;
*Spreadsheets
IDENTIFIERS *Mathematics of Money

ABSTRACT

The different approaches for solving problems and learning mathematics with technology are invaluable. This paper describes how to determine the variables of the ordinary annuity equation with a spreadsheet. Examples of future value of annuity, sinking fund annuity, the number of periods necessary for periodic payments plus interest to accumulate to a future value, and the rate per period needed for periodic payment plus interest to accumulate to a future value are described. With formulas, algebra, approximation, and technology, these mathematics applications are studied in different and meaningful ways. Although learning to use technology may be an obstacle to implementation for some educators, the use of spreadsheets enables the student to learn mathematics with little interference from technology and focus more on relevant topics and less on tedious calculations. Since society is relying more on technology, computer and calculator use for mathematics instruction is also becoming increasingly necessary. (Author)

SE

ED 469 079

PERMISSION TO REPRODUCE AND
DISSEMINATE THIS MATERIAL HAS
BEEN GRANTED BY

*J. Rochowicz
Jr.*

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)

1

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

☒ This document has been reproduced as
received from the person or organization
originating it.

☐ Minor changes have been made to
improve reproduction quality.

• Points of view or opinions stated in this
document do not necessarily represent
official OERI position or policy.

Calculating the Variables of

Finance on a Spreadsheet

Dr John A Rochowicz, Jr.

Alvernia College

Mathematics Department

SE 66824

Abstract

The different approaches for solving problems and learning mathematics with technology are invaluable. This paper describes how to determine the variables of the ordinary annuity equation with a spreadsheet. Examples of future value of annuity, sinking fund annuity, the number of periods necessary for periodic payments plus interest to accumulate to a future value, and the rate per period needed for periodic payments plus interest to accumulate to a future value are described. With formulas, algebra, approximation, and technology, these mathematics applications are studied in different and meaningful ways.

Although learning to use technology may be an obstacle to implementation for some educators, the use of spreadsheets enables the student to learn mathematics with little interference from technology and to focus more on relevant topics and less on tedious calculations.

Since society is relying more on technology, computer and calculator use for mathematics instruction is also becoming increasingly necessary.

Introduction

Computers and calculators can calculate complex expressions and graph mathematical relationships easily. Reform in mathematics continues to take place. Many students cannot do mathematics and exhibit anxiety toward mathematics. Lack of motivation, preparation and relevance are barriers to learning mathematics. With computers, the student learns practical mathematics and makes connections among many ideas. Through technology, students are able to consider concepts in greater depth.

Spreadsheets are technological tools that allow for experimentation and exploration of concepts where the computer-interface does not interfere with the learning of mathematics (Arganbright, 1985). Spreadsheets are ideal for studying mathematics because almost no programming is necessary and the algorithms required for successful problem solution can be conducted as done with pencil and paper.

The purpose of this paper is to illustrate various ways spreadsheets can be used for learning the mathematics of finance. Calculations of the ordinary annuity or future value of an annuity, sinking fund annuity, the time (or number of periods) needed for specified periodic payments plus interest to accumulate to some future amount, and the annual interest rate (or rate per period) needed for the periodic payments plus interest to accumulate to some future amount are discussed.

Through the use of spreadsheets, templates for various annuity calculations can be constructed. Once these templates are developed they can be applied to numerous other annuity applications. Text documentation of what is computed can always be incorporated right on the spreadsheet and the calculation of complex mathematical expressions can be performed with one keystroke.

Care in the entry of numbers and expressions in a technology using environment must be stressed where the order of operations is reinforced and studied. Building equations or formulas and seeing the calculations performed are learned. Checking results by handheld calculator to confirm and support the answers provided on the spreadsheet must be stressed. The student learns to apply one approach to solution to support another approach to solution. Learning becomes more self-paced, individual and independent.

Ordinary Annuity

An ordinary annuity is an accumulated sum of money including periodic payments made at the end of each period plus interest. Payments are made at a specified interest rate compounded for a specified number of times per year.

Consider the following: If you place \$100 into a fund at 6% compounded annually for 3 years, what is the amount of money accumulated at the end of the 3 years (payments plus interest). For this example the total number of periods is 3 times 1 or 3 periods and the rate per period is 6% divided by 1 or 6%. The calculated value is illustrated in table 1.

Insert Table 1 About Here

The accumulated amount is the sum of amounts at the end of each period in the table. The payment made for the first period accumulates the most interest since it is in the account the longest time and the final payment makes no interest (Roger, Haney & Laird, 1993). Notice that the amount expressions define a geometric series whose common ratio is $(1+i) = (1+0.06)$ which is greater than or equal to one. This geometric series does not converge and the nth partial sum is:

$$S_n = \frac{c(1-a^n)}{1-a}$$

where a is the common ratio $(1+i)$ and c is the common factor P in the expansion, that is,

$$\text{or } S = P \frac{1-(1+i)^n}{1-(1+i)} = P \frac{(1+i)^n - 1}{i}$$

defines the ordinary annuity equation where i is the rate per period, n is the number of periods needed for the money to accumulate, and P is the amount of each periodic payment. The derived formula is shown. Applying the ordinary annuity formula where $P = 100$, $i = .06$, and $n = 3$ the calculated accumulated amount is 318.36.

The next example applies the future value of an annuity formula as implemented on a spreadsheet.

Example 1

An investor deposits \$200 at the end of each month into a fund at 15% compounded monthly. Find the accumulated amount in the account at the end of 7.5 years. By handheld calculator, the following is computed using the future value of annuity equation

$$\frac{200 * ((1 + .15/12)^{(7.5 * 12)} - 1)}{(.15/12)} = 32491.00$$

A spreadsheet solution to this example is illustrated in Table 2.

Insert Table 2 About Here

Once the spreadsheet is constructed, various values can be entered in the different cells of the spreadsheet and results can be presented instantly. The dynamics of automatic calculations, interrelatedness between cells and student interaction with the spreadsheet are beneficial to student learning.

Table 3 provides a spreadsheet implementation of the ordinary annuity accumulation for each individual period of Example 1. Using the COPY command, calculations can be continued for each individual period as desired.

Insert Table 3 About Here

Sinking Fund

A sinking fund is the periodic payment needed to accumulate payment plus interest to a required amount at the end of a specified number of periods compounded for a specified number of times per year. Solving the ordinary annuity formula for P gives the sinking fund formula. The sinking fund formula is:

$$P = S \frac{i}{(1+i)^n - 1}$$

The following example illustrates a sinking fund annuity calculation.

Example 2

A business needs \$100,000 in 10 years. Find the periodic payments required to accumulate to \$100,000, if the interest is 18% compounded semiannually. Using a handheld calculator and the sinking fund formula, the following value is computed

$$\frac{100000(.18/12)}{((1+.18/12)^{(10*2)}-1)} = 1954.65$$

Table 4 presents a spreadsheet implementation for Example 2.

Insert Table 4 About Here

Similarly the periodic payment could be found as in Table 3 for each period.

Finding the number of periods

From the ordinary annuity equation

$$S = P \frac{(1+i)^n - 1}{i}$$

the number of periods n can be found by solving this equation for n using basic algebra. Divide both sides by P and multiply both sides by i to obtain

$$\frac{Si}{P} = (1+i)^n - 1.$$

Add 1 to both sides gives

$$\frac{Si}{P} + 1 = (1+i)^n$$

To solve for i take the natural logarithm of both sides and obtain

$$\ln\left(\frac{Si}{P} + 1\right) = n \ln(1+i)$$

Dividing both sides by $\ln(1+i)$ yields

$$n = \frac{\ln((Si/P)+1)}{\ln(1+i)}$$

An example which asks for the number of periods necessary for a periodic payment plus interest to accumulate to a certain amount is illustrated in Example 3.

Example 3.

Janet makes periodic payments of \$1000 into an account paying 9% compounded quarterly. If the accumulated amount (future value of the annuity) is \$25,000, find the number of periods necessary for the periodic payments plus interest to accumulate to \$25,000. Find the number of years it takes for these periodic payments to accumulate to \$25,000. (Note: If the total number of periods can be found then the number of years can be found by dividing the number of periods found by the number of periods per year.) Using the formula for n and a handheld calculator, compute

$$n = \frac{\ln(((25000 * .09/4)/1000)+1)}{\ln(1+.09/4)}$$

The result is about 20 periods. Since the interest is compounded quarterly, the number of years it takes to grow to 25000 is $20/4$ or 5 years. A spreadsheet solution to Example 3 is illustrated in Table 5.

Insert Table 5 About Here

Table 5 demonstrates the spreadsheet calculations for the number of periods (and number of years) for periodic payments plus interest to accumulate to a specified amount.

Finding the rate per period i

Finding the rate per period for periodic payments to accumulate to a specified amount after compounding a number of times a year for a certain number of years is not as easy to calculate as the other variables in the ordinary annuity equation including accumulated amount (S), periodic payments (P), and the number of periods (n). Example 4 illustrates how to calculate i .

Example 4.

Determine the least interest rate necessary for an investment of \$350 compounded each quarter to accumulate to \$150,000 after 25 years.

Using the ordinary annuity equation and solving for i is not possible in closed form. In order to determine this interest rate apply the ordinary annuity equation to obtain a polynomial function in i , namely $f(i) = P(1+i)^n - S - P$. A zero of this function on the interval from 0 to 1 can be found by various approximation techniques.

A graphing utility or a root finding method such as Newton's Method (Larson, Hostetler & Edwards, 2002) can be used to locate the zeroes of the function. Table 6 implements Newton's Method on a spreadsheet to find the interest rate per period for Example 4.

Insert Table 6 About Here

Using the COPY command and copying the formulas in cells A30, B30, C30, D30, and E30 to A37, B37, C37, D37, and E37 provides the root for Example 4 where an initial guess was 0.05.

The least rate per period for \$350 to accumulate to \$150000 in 25 years if the payments are compounded quarterly is about 2.49%. The annual interest rate is then 4 times 0.024861 or about $0.099444 = 9.94\%$.

A variety of technological ways other than spreadsheets can be used to obtain the approximate root of 0.024861. The use of a computer algebra system such as DERIVE and using the SOLVE command provides a solution to the equation defined by Example 4 (that is, solve $350(1+i)^{100} - 150,000(i) - 350 = 0$). Implementation of a software program written in PASCAL, FORTRAN or BASIC for Newton's Method is also an easy way to find the root of the equation between 0 and 1. Also, the use of any graphing utility and repeated use of the ZOOM IN capability locates the point where the graph crosses the x axis.

Amortization and the present value of annuity (variations of the ordinary annuity equation) can be similarly calculated using a spreadsheet. See Rogers, Haney & Laird (1993) and Lial & Hungerford (1999) for definitions and discussions of amortization and present value of an annuity.

Conclusion

Using spreadsheets for financial mathematics allows the student to connect ideas and interpret the reasonableness of answers. From confirming results by handheld calculator, practicing the order of operations, solving a variety of algebraic equations including logarithmic or exponential equations and finding zeroes or roots of functions, the student is studying relevant and useful mathematics. Focus in learning mathematics shifts from tedious computation to thinking about solutions.

If the student learns to calculate annuity values by using technological tools such as the spreadsheet, the use of tables can be reduced. Templates can be produced where a change in various cells on the spreadsheet calculates a variety of finance problems automatically. Through the use of technology, algebra, and the ordinary annuity formula meanings are given to S (ordinary annuity accumulated amount), P (sinking fund annuity payment), n (total number of periods), and i (rate per period). Numerous textbooks in business and finite mathematics present calculations that involve accumulated amount, periodic payments for various annuity equations including future value, present value, and amortization. Topics such as finding the number of periods for payments to grow to an accumulated amount and the rate per period for periodic payments to grow to an accumulated amount are not readily found. With spreadsheets these additional topics can be studied and learned. Technology enables the student to study the ordinary annuity equation and make sense of all four variables involved. Skills that are mindful applications of mathematics and not the mindless manipulation of meaningless mathematical symbols are being acquired.

References

- Arganbright, D. E. (1985). *Mathematical Applications of Electronic Spreadsheets*. New York, NY: McGraw-Hill Book Company.
- Larson, R. E., Hostetler, R. P. & Edwards, B. H. (2002). *Calculus* (7th Edition). Lexington, MA: D. C. Heath and Company.
- Lial, M. L., Hungerford, T. W. (1999). *Mathematics With Applications* (7th Edition). Reading, MA: Addison-Wesley Longman, Inc.
- Rogers, J. E., Haney, B. F., & Laird, D. (1993). *Fundamentals of Business Mathematics*. Boston, MA: PWS-KENT Publishing Company.

Table 1.

Ordinary Annuity Definition

Period	Payment	Amount	
1	100	$100(1+.06)^2$	= 112.36
2	100	$100(1+.06)$	= 106.00
3	100	100	= 100.00
Accumulated Amount			= 318.36

Note. $S = \frac{100 [1.06^3 - 1]}{.06} = 318.36$

Table 2.

Calculating Ordinary Annuities

Ordinary Annuity (Future Value of an Annuity)

	C	D	E	F
4 Periodic Payment P	200	Periods per Year		12
6 Interest Rate	0.15	Number of Years		7.5
8 Rate per Period (i)	0.0125	Number of Periods (n)		90

Ordinary Annuity

Future Value $S = P * ((1+i)^{(n)} - 1) / i$

15 $S = 32941.00$

Note. Necessary cell formulas are:

in C8 - +C6/F4

in F8 - +F4*F6

in E15 - +C4*((1+C8)^(+F8)-1)/+C8)

Table 3.

Calculating Ordinary Annuities for Each Period

	C		F
4 Periodic Payment P	200	Periods per Year	12
6 Interest Rate	0.15	Number of Years	7.5
8 Rate per Period (i)	0.0125	Number of Periods (n)	90
A	B		
n	S		
	Ordinary		
	Annuity		
	(Future)		
18	1	200	
19	2	402.5	
20	3	607.5312	
21	4	815.1253	
22	5	1025.314	
23	6	1238.130	
.	.	.	
.	.	.	
104	87	31150.65	
105	88	31740.04	
106	89	32336.79	

Table 3 ... continued

107 90 32941.00

Note. Necessary cell formulas are:

in C8 - +C6/+F4

in F6 - +F4*+F6

in A19 - +A18+1

in B19 - + $\$C\$4*((1+\$C\$8)^{(A19)-1})/\$C\$8$

COPYING cells A19 and B19 down to A107 and B107 gives the result of 32941.00

Table 4.

Sinking Funds

	C	D	E	F
4 Future Value	100000	Periods per Year		2
6 Interest Rate	0.18	Number of Years		10
8 Rate per Period i	0.09	Number of Periods (n)		20
Periodic Payment		$P = S * i / ((1+i)^{(n)} - 1)$		
13	P =	1954.65		

Note. Necessary cell formulas are:

in C8 - +C6/F4

in F8 - +F4*F6

in E13 - +C4*C8/((1+C8)^(F8)-1)

Table 5.

Finding the Number of Periods

	C		F
4 Periodic Payment (P)	1000	Periods per Year	4
6 Interest Rate	0.09	Future Value (S)	25000
8 Rate per Period I	0.0225		
Future Value $S = P * ((1+i)^n - 1) / i$			
	$n = @ln((S/P)+1) / @ln(1+i)$		
17	n = 20.1 periods		
19	T = 5 years		

Note. Necessary cell formulas are:

in C8 - +C6/+F4

in C17 - @LN((F6*C8)/C4)+1)/@LN(1+C8)

in C19 - +C17/F4

Table 6.

Finding the Rate per Period i

A	B	C	F	H
6	Periodic Payments (P)	350	Periods per Year	4
8			Number of Years	25
10	Future Value (S)	150000	Number of Periods	100

Finding a zero of the function in i:

B	D	F	H
15	$f(i) = 350 \cdot (1+i)^{(100)} - 150000 \cdot i - 350$		
17	$f(i) = 35000 \cdot (1+i)^{(99)} - 150000$		

Newton's Method for $f(i)$ where $0 < i < 1$

24 Initial Guess for i: 0.05

	A	B	C	D	E
	Iteration	I	$f(i)$	$f'(i)$	
29	1	0.05	38175.44	4233375.	not yet
30	2	0.040982	12929.45	1716198.	not yet
31	3	0.033448	4028.912	759207.0	not yet
32	4	0.028141	1043.969	396153.5	not yet
33	5	0.025506	168.2220	273614.6	not yet
34	6	0.024891	7.579835	249196.4	not yet
35	7	0.024861	0.017820	248025.2	0.024861

Table 6 ...continued

36	8	0.024861	0.000000	248022.4	0.024861
37	9	0.024861	-8.6E-13	248022.4	0.024861

Note. Necessary cell formulas are:

in F10 - +F6*F8

in B15 - +C6

in B17 - +C6*C8

in D15 - +F10

in D17 - +F10-1

in F15 - +C10

in F17 - +C10

in H15 - +C6

in A29 - 1

in B29 - +D24

in C29 - +\$C\$6*(1+B29)^(F\$10)-\$C\$10*B29-\$C\$6

in D29 - +\$C\$6*F\$10*(1+B29)^(F\$10-1)-\$C\$10

in E29 - @IF(@ABS(B30-B29)<1*10^(-6),B30,"not yet")

in A30 - +A29+1

in B30 - +B29-(C29/D29)

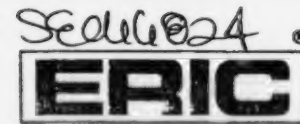
in C30 - +\$C\$6*(1+B30)^(F\$10)-\$C\$10*B30-\$C\$6

in D30 - +\$C\$6*F\$10*(1+B30)^(F\$10-1)-\$C\$10

in E30 - @IF(@ABS(B31-B30)<1*10^(-6),B31,"not yet")



U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)



REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Calculating the Variables of Finance on a Spreadsheet	
Author(s): John A. Rochowicz, Jr EdD	
Corporate Source:	Publication Date:

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

Level 1

☒

Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) and paper copy.

The sample sticker shown below will be affixed to all Level 2A documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2A

Level 2A

☐

Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only

The sample sticker shown below will be affixed to all Level 2B documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2B

Level 2B

☐

Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.

If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign
here,→
please

Signature: Dr. John A. Rochowicz, Jr	Printed Name/Position/Title: John A. Rochowicz, Jr Professor of Mathematics
Organization/Address: Alvernia College 400 St. Bernardine St Reading, PA 19607	Telephone: 610-796-8257 E-Mail Address: John.Rochowicz@Alvernia.edu
	FAX: 610-796-8241 Date: 8/19/2002

(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:



However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
4483-A Forbes Boulevard
Lanham, Maryland 20706
Telephone: 301-552-4200
Toll Free: 800-799-3742
FAX: 301-552-4700
email: info@ericfac.piccard.csc.com
WWW: <http://ericfacility.org>